SUBJECTIVE REFRACTION: A NEW VECTORIAL METHOD FOR DETERMINING THE CYLINDER (1/3)

The refraction technique traditionally used to determine the corrective cylinder for a prescription has changed very little over the years, mainly due to the limitations imposed by subjective phoropters, which present lenses in increments usually no smaller than 0.25 D.
Today, thanks to phoropters with continuous power changes that allow to simultaneously and accurately act on sphere, cylinder and axis, it is now possible to develop new refraction techniques.
This series of three articles describes the principles of a new vectorial method for determining the corrective cylinder and presents the rationale for an associated automated cylinder search algorithm.

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KEYWORDS
Subjective refraction, vectorial refraction, dioptric space, cylinder search, cross cylinders, phoropter, refraction algorithm, Vision-R™ 800.

For nearly a century, the refraction technique used to determine a patient's corrective cylinder has remained almost totally unchanged, mainly because subjective phoropters themselves have changed very little. Practitioners generally use the Jackson cross-cylinder method, studying the variation of its effects for different positions, to determine first the cylinder axis, then the cylinder power and, finally, to adjust the effect on sphere power. With a subjective phoropter, practitioners present spherical and cylindrical lenses in front of the patient's eye in increments usually not smaller than 0.25 D and 5 degrees in axis. Simultaneous action on the sphere, cylinder and axis is also not possible.

Today, the advent of phoropters that offer continuous power changes – with a resolution of 0.01 diopter and 0.1 degree – and allow to act on sphere, cylinder and axis all at the same time[*] makes a new approach to subjective refraction possible: it is called "Digital Infinite Refraction™"[1]. A vectorial method has been developed to determine the cylinder that is both more consistent and more accurate.

This series of three articles provides an overview of this new vectorial method. In this first article, we will review the vectorial definition of refraction and its representation in the 'Dioptric Space' before offering a general comparison of the "Traditional Refraction" and "Digital Infinite Refraction™" methods. The second article will describe in detail the techniques used in "Traditional Refraction" and "Digital Infinite Refraction™" to determine cylinder axis and cylinder power. The third and final article will present the new method of determining the cylinder made possible by "Digital Infinite Refraction™" in comparison with to the "Traditional Refraction" method, and will discuss its application to the development of an automated algorithm for determining the cylinder.

Read on to learn more about this new vectorial method for determining the corrective cylinder. Please note you will need to be familiar with the basic principles of refraction to fully understand these articles.

(*) Vision-R™ 800 phoropters with continuous power changes, Essilor Instruments"

Vectorial representation of the cylinder in a dioptric space

"Polar" vs "Cartesian" expression of a refraction:

Although in ophthalmic optics, the formula of a refraction is traditionally expressed with reference to its "Polar" expression (sphere, cylinder and axis), it is also possible to give it a "Cartesian" expression in the form of three coordinates:

1) the spherical equivalent or Mean sphere *M*, equal to the sphere power augmented by half of the cylinder power,
2) the cylinder component along the horizontal axis at 0° (J0°), representing the direct/indirect component of astigmatism,
3) the oblique component of the cylinder along the oblique axis at 45° (J45°), representing the oblique component of astigmatism.

The advantage of this cartesian expression is that it expresses the refractive formula in the form of three independent components, themselves expressed in a single and consistent unit: diopters. These can effectively replace the components of the traditional polar expression of a refraction (sphere, cylinder and axis), which are interdependent and expressed in different units: diopters for sphere and cylinder and degrees for the axis. The cartesian expression yields a unique global formula for a refraction that facilitates its analysis and statistical comparisons.[2]

By way of illustration, Table 1 shows examples of refraction formulas expressed in traditional polar coordinates transposed into cartesian coordinates. We can see that the cartesian expression of a refractive formula involves expressing the refraction in the form of an average component and two pure cylindrical components, which is to say similar to Jackson cross-cylinder formulas with null mean sphere power, one of them at 0°/90°, representing the horizontal/vertical component of the astigmatism, and the other at 45°/135°, representing its oblique component.

The relationship between the polar and cartesian expressions of a single refraction formula is based on a simple trigonometry calculation. It is relatively easy to move from one expression to the other:

– If we know the traditional polar formula of a refraction Sph (Cyl) Axis, we can calculate the three coordinates of its Cartesian expression using the following formulas:
  - $M = Sph + Cyl / 2$;
  - $J0° = Cyl * \cos(2 * Axis)$;
  - $J45° = Cyl * \sin(2 * Axis)$.

Because of the non-trigonometric cycle of the axis (its variation from 0 to 180° rather than 0° to 360°), it is necessary to double the value of the cylinder axis.

– Inversely, if we know the cylinder's cartesian components, J0° and J45°, it is easy to determine its polar (cylinder and axis) components via vectorial composition. And for the sphere, all we need to do to find its value is algebraically subtract half of the cylinder's value from that of the spherical equivalent. The formulas are as follows, using a negative cylinder convention:
  - $Sph = M - Cyl / 2$
  - $Cyl = -\sqrt{J0°^2 + J45°^2}$
  - $Axis = 0.5 * \mathrm{ArcTan}\,(J45° / J0°) + C$, with C constant equal to 90 if J0° > 0 and equal to 0 if J0° < 0.

To make it easier to grasp and simpler to represent visually, we have opted in this article not to keep the ½ weighting between the values of the J0° and J45° components, on the one hand, and the *M* spherical equivalent power on the other hand, as is generally the case in the literature on vectorial expressions of refraction. The principle remains the same but this simplification is more readily understandable.

Representation of a prescription in a "Dioptric Space":

The advantage of the cartesian expression of a refraction is that it can represent any refractive formula in a three-dimensional orthogonal system called the "Dioptric Space". Any prescription is represented in it by a unique vector whose projections on the system three axes are the cartesian coordinates of the refractive formula.
As a result, the following is shown on the three axes:
– the spherical equivalent power, or mean sphere *M*,
– the horizontal component of the cylinder J0°,
– the oblique component of the cylinder J45°.

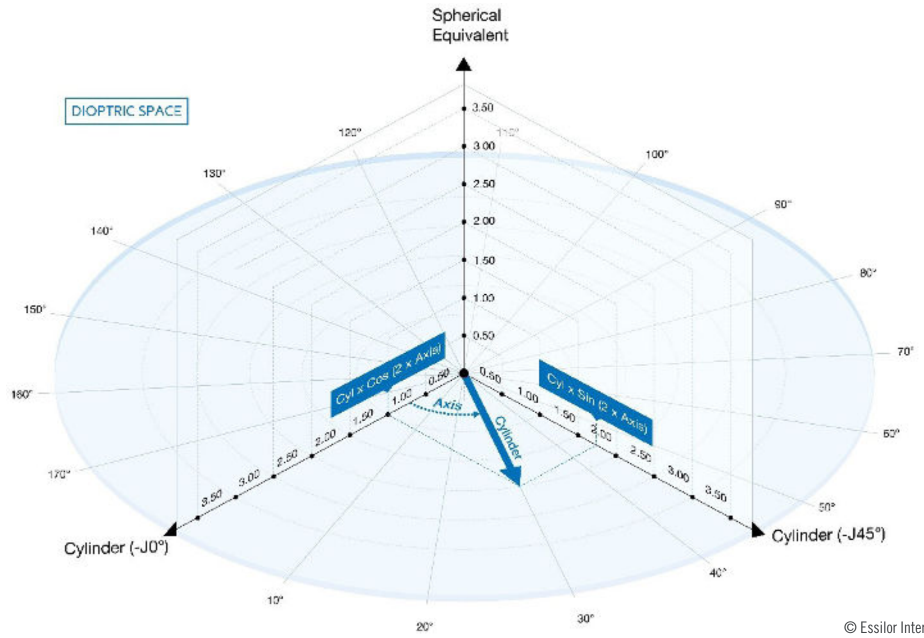
Table 1: Polar and Cartesian expressions of various refraction formulas

POLAR EXPRESSION			CARTESIAN EXPRESSION		
Sphere	**Cylinder**	**Axis**	**M**	**J0°**	**J45°**
+2.00			+2.00	0.00	0.00
-2.00			-2.00	0.00	0.00
Plano	-2.00	0	-1.00	-2.00	0.00
Plano	-2.00	90	-1.00	+2.00	0.00
Plano	-2.00	45	-1.00	0.00	-2.00
Plano	-2.00	135	-1.00	0.00	+2.00
+1.00	-2.00	120	0.00	+1.00	+1.73
+1.00	-2.00	30	0.00	-1.00	-1.73

The three-dimensional representation of the dioptric space we use, which allows us to easily visualise the characteristics of any refractive formula in 3D (see Figure 1), is a modified version of the conventional representation explained in more detail in various reference publications.[2,3,4,5] The sphere is expressed along the vertical axis and the cylinder along the horizontal plane : the cylinder axis is represented by the rotation around the vertical axis and the cylinder power by the distance from the origin, here chosen according to the negative cylinder convention. This model can be used to simply depict any refractive formula in the form of a single vector in the space and to study its variations during a refraction examination: the purpose of "Vectorial Refraction".

Figure 1: Vectorial representation of refraction in a dioptric space.

a) Cartesian coordinates: example of a refraction formula of +1.00 (-2.00) 30°



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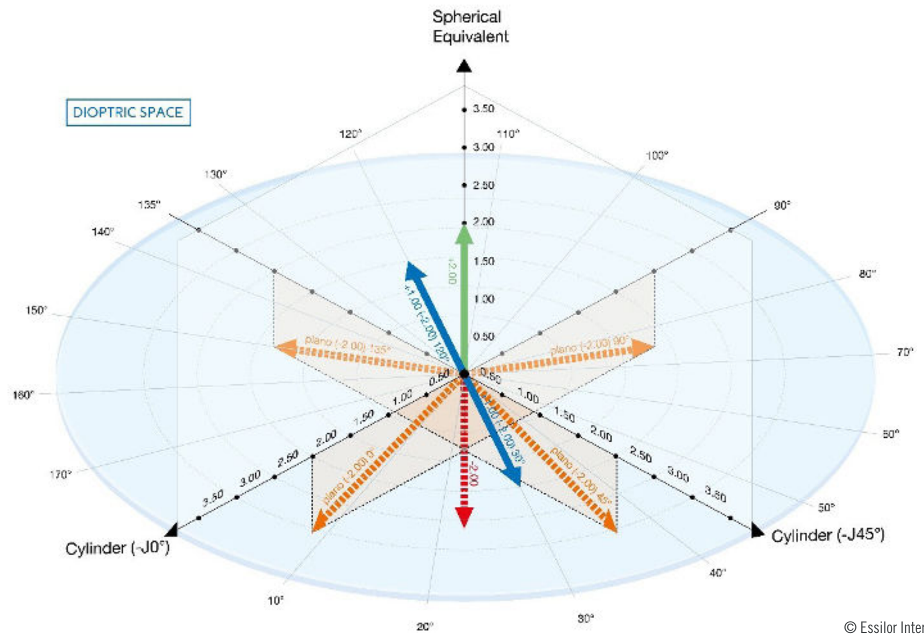
b) Examples of vector representations of different refraction formulas (presented in Table 1):



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Sphere formulas: +2.00 (in green) and -2.00 (in red);
Astigmatic formulas: plano (-2.00) with cylinder axes of 0°, 45°, 90° and 135° (in orange)
and +1.00 (-2.00) with cylinder axes at 30 ° and 120 ° (in blue).

The example we will be using in the rest of this article, a refraction formula of +1.00 (- 2.00) 30° with a null spherical equivalent, was chosen for the convenience of the graphic representations, since the corresponding vector is located on the horizontal J0°/J45° plane. For any other refraction whose spherical equivalent power is not null, the approach would be the same but the vector would move in the space, leaving a trace identical to that made on the J0°/J45° plane but on a parallel horizontal plane, corresponding to the value of the spherical equivalent.

Traditional refraction vs Digital Infinite Refraction™: similarities and differences

Although the traditional and digital refraction techniques have a few principles in common, they differ greatly in other points. Let us take a look at these similarities and differences before examining them more closely in the following two articles.

Refraction with "presentation of lenses" vs refraction with "continuous power changes"

– The "traditional" refraction technique involves presenting spherical and cylindrical lenses in front of the patient's eye. This can be done with trial frames and trial lenses, using a manual phoropter with mechanical lens changes or an automated phoropter with motorised lens changes. Regardless of the instrument used, the method involves presenting lenses in 0.25 D increments; only the way the lenses are changed is different. Furthermore, the sphere power, cylinder axis and cylinder power must be examined separately, one after the other, during the examination.

– The "digital" technique, on the other hand, takes advantage of the capacities of an optical module with continuous power changes[*] controlled by micro-motors with digital commands. This technology allows to switch instantly from one optical formula to another by modifying the optical powers and using the variation increment desired (with a resolution of 0.01 D). It is also possible to change the sphere power, cylinder axis and cylinder power simultaneously, allowing to move from one corrective formula to another with no delay. This property is what makes the new refraction technique possible.

Determining the refraction components "successively" vs "simultaneously"

– "Traditional" refraction techniques involve first determining the sphere and then the cylinder axis and power before finally adjusting the sphere. For the cylinder determination, it is important to always start with the cylinder axis before moving on to the cylinder power, otherwise the latter value will be impossible to determine correctly. While it is possible to adjust and find the correct value of a cylinder axis if its starting power is not correct, adjusting the power of a cylinder with an incorrect starting axis leads to a value different from that which would have been obtained with the correct axis.

– In the "digital" refraction technique, we firstly look for the mean sphere and then, in the same sequence, move on to the cylinder power and axis, keeping the spherical equivalent power exactly constant with a resolution of 0.01 D. Two refraction components are considered here: a power component along the initial axis of the starting correction and an axis component that is perpendicular to the latest in the dioptric space. Since these power and axis components are orthogonal and independent of each other, cylinder seeking can begin with either the axis or the power component. That said, the initial refraction measurements provided by autorefractometers are generally more accurate in the axis value than in the power value. This is why cylinder power is the starting point for the new digital refraction technique, unlike the traditional method which begins with seeking the axis.

Determining astigmatism: "physical" vs "virtual" cross cylinders

Both cylinder determination techniques ("traditional" and digital refraction) use the Jackson cross-cylinder method, named after the American ophthalmologist who developed it in the early 20th century.
Remember that the cross cylinder is a spherical-cylindrical lens resulting from a combination of two plano-cylindrical lenses with identical powers but opposite signs positioned perpendicularly to each other (this is the reason for the name "cross cylinders") and with a null spherical equivalent. Determining the corrective cylinder involves placing the cross cylinder in front of the patient's eye while they are wearing their correction and studying the variations in the sharpness of the patient's vision that result from the combination of the residual astigmatism of the eye + lens system and that of the cross cylinder at different positions.

Although this cross-cylinder method is similar in both refraction techniques, the approaches used are very different.

– In traditional refraction, physical cross cylinders in the phoropter are flipped over during the examination. Cross cylinders of +/-0.25 D or +/-0.50 D are generally used; their respective optical formulas are +0.25 (-0.50) and +0.50 (-1.00). Due to its construction, the "handle" of any cross cylinder bisects the axes of its positive and negative cylinders in such a way that, by simply flipping them over, one can switch their positions or, in other words, instantaneously turn the axis of the cross cylinder by 90° without modifying the mean sphere value. Practitioners use this property to look for the cylinder axis and power, seeking the orientation of the axis and then the value of the power at which turning over the cross cylinder produces an identical blurred vision for the patient. We will look at this technique in more detail in article two.

– In "digital" refraction, an optical principle similar to the Jackson cross-cylinder method is used but no cross cylinders are physically present in the phoropter. Optical cross-cylinder effects are generated in the optical module using calculations in combination with the existing correction. There is therefore no positioning of a cross cylinder in front of the patient's eye nor any interruption in their vision during the switch, only seamless changes in optical correction that the patient perceives instantly. The cross-cylinder power is not limited to that of a traditional cross cylinder (of +/- 0.25 D or +/- 0.50 D) but can be chosen with a resolution of 0.01 D to allow for easy comparison between the two positions and configuration during the design of the cylinder determination algorithm. It could also be adjusted during the refraction examination according to the patient's sensitivity. This flexibility offers remarkable possibilities in terms of improving and adapting refraction methods. In the example we have been using in this article, the cross-cylinder power is +/- 0.35 D.

Later, in article two, we will examine in detail the practical implementation and differences of these techniques when it comes to determining the cylinder.

An "unchangeable" traditional technique vs an "upgradeable" digital technique

– In "traditional" refraction, the testing technique and method for determining the cylinder have remained the same for the past century and there is little room for any change due to the physical limitations and mechanical constraints imposed by the instruments. The refraction is entrusted entirely to practitioners, who apply the knowledge they have acquired, their experiences and the type of approach they have chosen. As a result, there are inevitably variations among refraction results.

– In "digital" refraction, on the other hand, the testing and refraction methods used are innovative and upgradeable. Because the optical module is controlled by calculations and totally flexible, a wide field of possibilities opens up for the development of new refraction methods. The first refraction determination assistance algorithms have been invented to formalise the first examination principles. They should be able to bring about a certain standardisation in refraction methods. These algorithms are already "adaptive" that is, they have the capability to adapt to patients' answers during the examination itself. They will undoubtedly be improved upon as advances in this area are made, making many refraction assistance solutions possible in the future. The new "Digital Infinite Refraction™" approach therefore holds considerable potential for ongoing improvements in refraction methods.

We will continue the presentation and discussion of this topic in two articles to come.

KEY INFORMATION:

- The cylinder search technique has changed very little since Jackson's invention of the "cross-cylinder" method in the early 20th century because subjective phoropters whose functioning is based on a presentation of various lenses have themselves not changed much.
- Today, with the advent of phoropters offering continuous power changes, it is now possible to offer a new cylinder search method based on a vectorial approach to refraction.
- This method explores the "dioptric space" in a more direct way, searching for the cylinder power and axis simultaneously while keeping the spherical equivalent power exactly constant.
- Combined with the properties of a very precisely controlled optical module that is integrated into refraction search algorithms, this new technique offers great scope for advancements in refraction methods.

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